Democracy as a Critical System: Formal Methods for Voting

Joseph Kiniry
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novel,
difficult
topics

basic research ⟷ applied research

Research Style

open source
tools usable by
undergraduates

mathematics
biased toward
usable tools

seamless
architectures

self-healing
systems

Current Research Projects

verified
elections

specification and
verification of
concurrent Java

formal methods for
sensor networks

Formal Methods

“Pure”
Formal Methods

“Applied” Formal Methods

Critical Systems

military

biomedical

avionics

automotive

Critical Systems

financial

aeronautics

nuclear

transport

voter registration

voter trust

government legitimacy

voting systems

Democracy

voting schemes

casting ballots

election outcomes

counting ballots

society

troublemaker

impact

Activism and Science

good

education

obligation

Computer Scientist

Mathematician

Voting Theory

Computers in Elections

Electronic Voting

e-Voting

punchcard
ballots

mechanical
ballot boxes

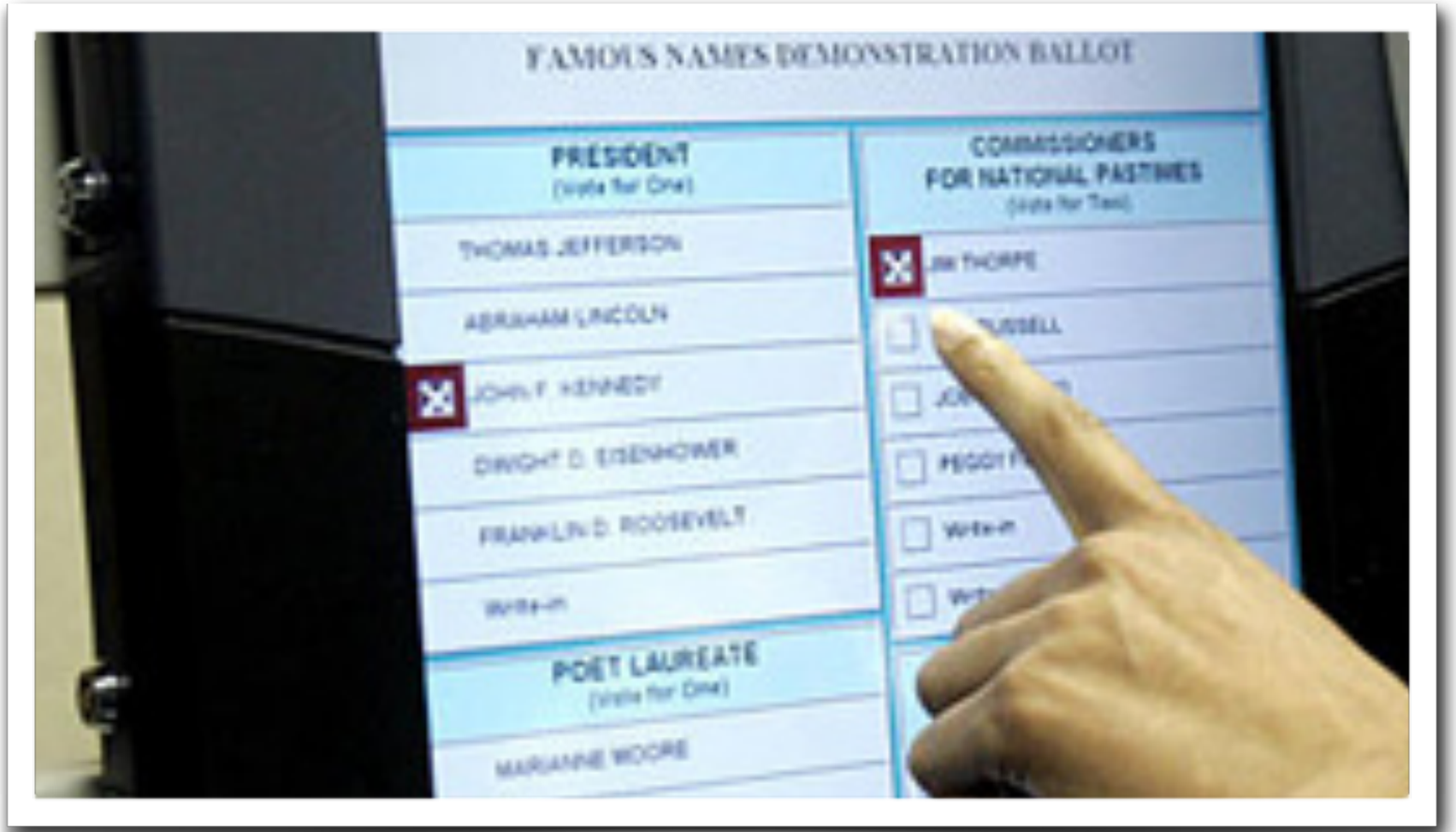
physical
locks

Voting Machines

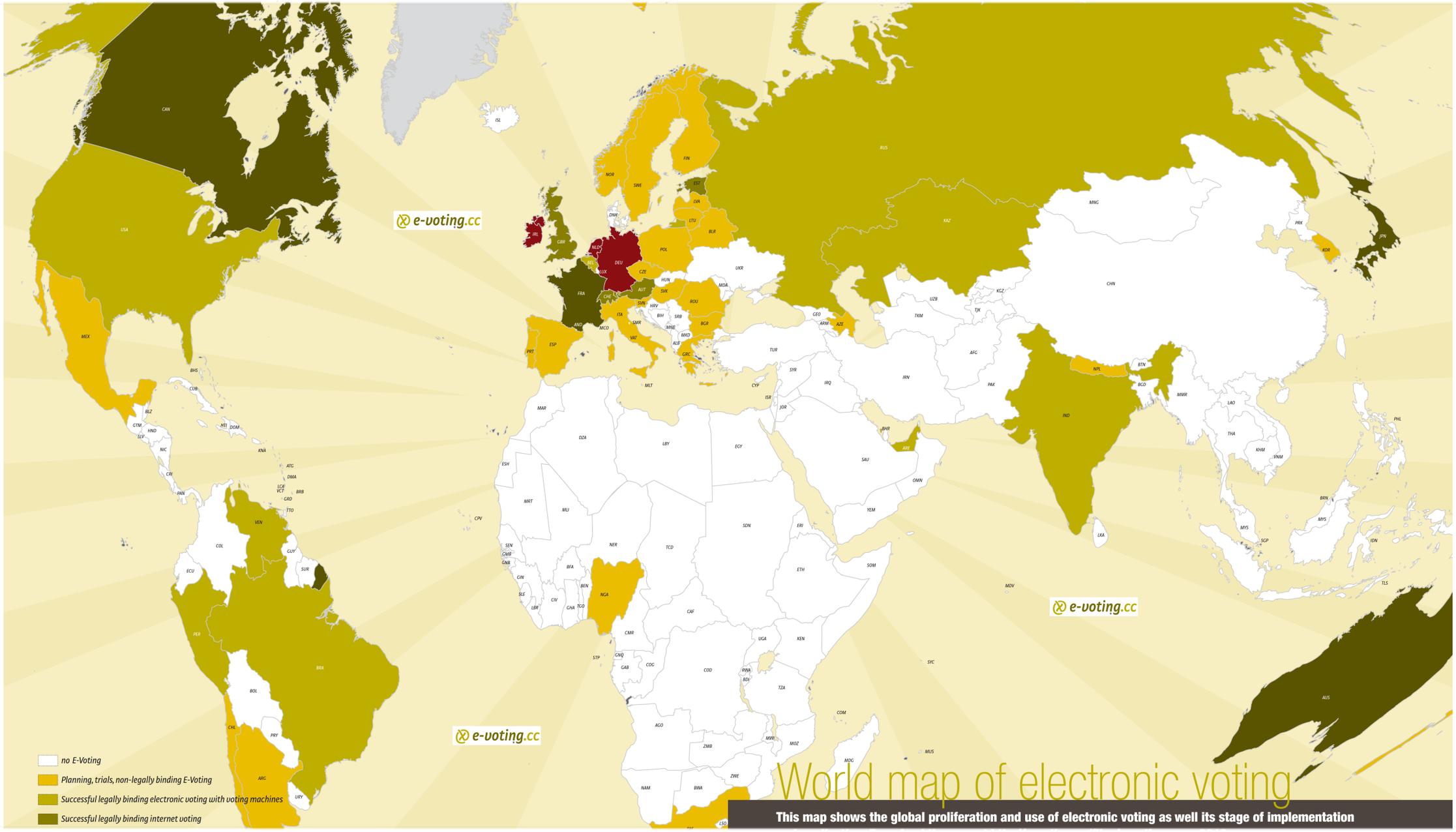
dedicated
primitive
hardware

off-the-shelf
Windows
machines

lever
machines



e-Voting Worldwide



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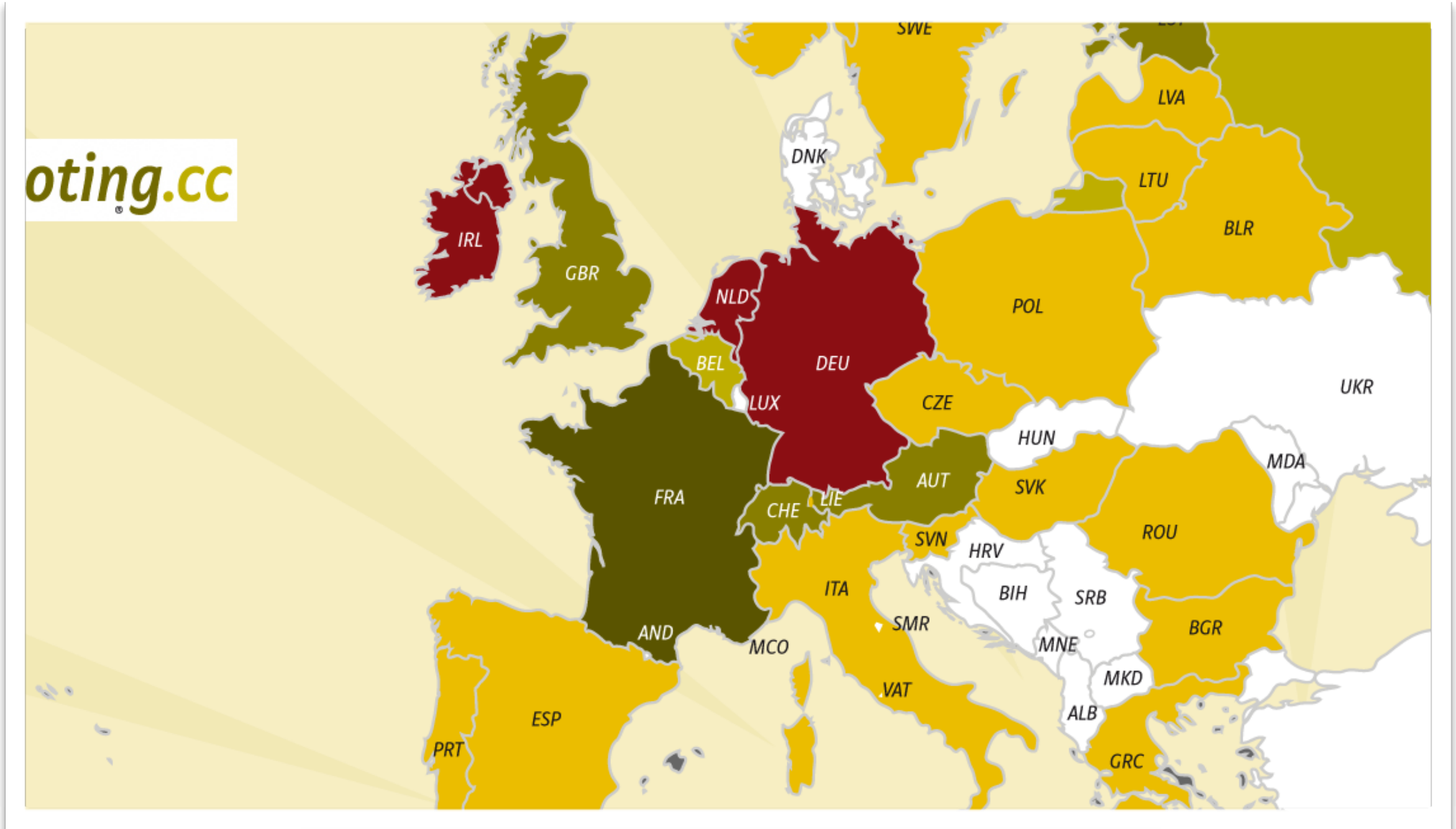

e-Voting in the EU

dedicated computer-based voting machines since the late 90s

people generally trust the government

experiments in remote voting for expats

Computer-based Voting in The Netherlands

hacking an election

tally system developed with formal methods

recommendations to the government

KOA

PR-STV

novel social vote counting

last-minute secret purchase of €40M in Nedap machines

Computer-based Voting in Ireland

PowerVote

Vótáil

independent system testing

CEV

scrapping e-voting at a cost of €55M

people generally trust the government

claim: no computers are used in voting

in truth: closed-source tally system used to compute final outcome

Computer-based Voting in Denmark

regular proposals to introduce e-voting

DVT

e-voting trials at the local level

DemTech

experiences with open source e-voting systems

experiences with proprietary e-voting systems

Experiences in Hacking Voting Systems

hacking remote elections

hacking kiosk-based voting computers

analyzing academic voting systems

most open source voting systems are not tested

most proprietary voting systems are not tested

Testing Voting Systems

"hard-core" testing is random testing of multiple implementations

random testing is no testing

how does one rigorously test a voting system?

Relating The Law to Software

The State of e-Voting Software Today

Table 1 gives – as an example – the numbers from the multi-member constituency of Østjylland (Eastern Jutland).

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This step determines which parties are eligible for compensatory seats. This is done by checking if participating parties meet any of three requirements. Thus, the Danish electoral system has not one, but three different electoral thresholds, and parties qualify for participation in the allocation of compensatory seats by any one of them. The three thresholds are:

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The Parties' Votes:				
K. Christian People's Party	30,013	5,513	7,635	16,865
Y. New Alliance	97,295	40,241	30,358	26,696

The Law

```
char*M,A,Z,E=40,J[40],T[40];main(C){for(*J=A=scanf(M="%d",&C);
--          E;          J[          E]          =T
[E   ]=  E)   printf("._");  for(;(A-=Z=!Z)  ||  (printf("\n|"
)    ,   A    =          39          ,C          --
)    ;   Z    ||    printf   (M   ))M[Z]=Z[A-(E   =A[J-Z])&&!C
&    A   ==          T[                          A]
|6<<27<rand()||!C&!Z?J[T[E]=T[A]]=E,J[T[A]=A-Z]=A,"_.":" |"];}
```

e-Voting Software

$\emptyset$

Refinement Relation

In our tests, it counts correctly.

Overall Correctness Argument

Trust us, it works.

How hard can it be, adding one over and over?

The State of *Verified* e-Voting Software Today

concept analysis

invariants

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The Law

Informal EBON

```
class_chart LOGICAL_CLOCK

explanation
  "A logical clock."
query
  "What is the current time for this clock?"
command
  "Advance the clock; update the clock's time."
constraint
  "The time must be non-negative.",
  "Must support concurrent use by multiple clients."
end
```

Formal EBON

```
indexing
  about:        "A logical clock.";
  title:        "TickTockClock";
  author:       "Joe Kiniry";
  copyright:    "Copyright (C) 2007 Joe Kiniry";
  organisation: "School of Computer Science and Informatics, UCD";
  date:         "January 2007";
  version:      "Revision: 11";

static_diagram
component
  deferred class LOGICAL_CLOCK

  feature
    my_time: INTEGER  -- The current time of this clock.

    -- What is the current time of this clock?
    deferred get_logical_time: INTEGER
      -- concurrency: CONCURRENT
      -- modifies: QUERY
      ensure
        Result = my_time;
      end

    deferred advance  -- Advance this clock's time.
      -- concurrency: GUARDED
      -- modifies: my_time
      ensure
        -- This clock's time has monotonically increased.
        old my_time < my_time;
    end

  invariant
    0 <= my_time;

  end -- class LOGICAL_CLOCK

end --component
```

JML

```
/**
 * A logical clock.
 * @title         "TickTockClock"
 * @date          "2007/01/23 18:00:49"
 * @author        "Fintan Fairmichael"
 * @organisation  "CSI School, UCD"
 * @copyright     "Copyright (C) 2007 UCD"
 * @version       "$ Revision: 1.7 $"
 */
public interface LogicalClock {
  // The current time of this clock.
  //@ public model instance \bigint _time;

  //@ public invariant 0 <= _time;

  /**
   * @return What is the current time of this clock?
   * @concurrency CONCURRENT
   */
  //@ ensures \result == _time;
  public /*@ pure @*/ long getLogicalTime();

  /**
   * Advance this clock's time.
   * @concurrency GUARDED
   */
  //@ assignable _time;
  //@ ensures \old(_time) < _time;
  //@ ensures (* _time has been increased. *);
  public void advance();
}
```

Java

```
/**
 * A logical clock implementation.
 * @author "Joseph Kiniry"
 */
public class LogicalClockImpl implements LogicalClock {
  /** The current logical time. */
  private long my_time = 0; //@ in _time;
  //@ private represents _time <- my_time;

  public long getLogicalTime() {
    return my_time;
  }

  public void advance() {
    my_time++;
  }
}
```

e-Voting Software

Danish Law

Table 1 gives - as an example - the numbers from the multi-member constituency of Ribtjerland (Eastern Jutland).

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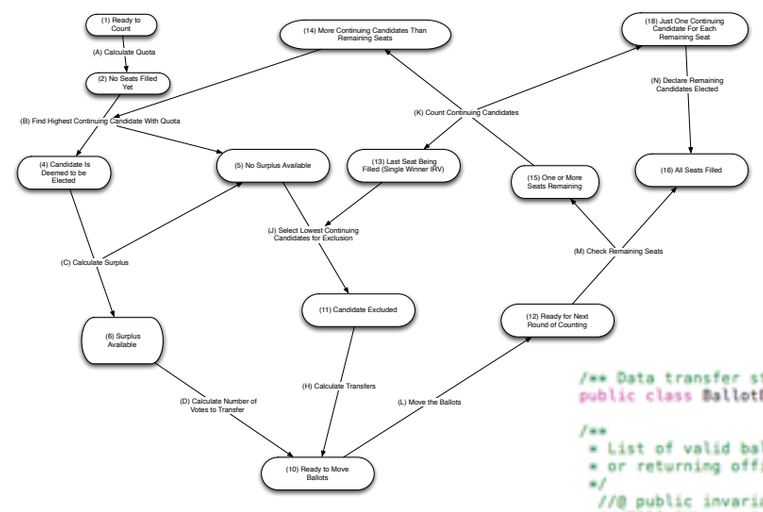
Threshold 2:				
Valid votes per multi-member constituency seat	n.a.	26,908	25,103	25,146
Threshold 3:				
2 per cent of valid national votes	66,189	-	-	-
The Parties' Votes:				
X: Christian Peoples Party	30,013	5,513	7,635	16,865
Y: New Alliance	97,295	40,241	30,358	28,696

Verified Software




```
/** Data transfer structure for set of all valid ballots */
public class BallotBox {

/**
 * List of valid ballot papers, already shuffled and mixed by the data loader
 * or returning officer.
 */
  //@ public invariant \nonnullelements (ballots);
  // TODO JML warning: array nullity is invariant for assignment
  protected /*@ non_null spec_public @*/ Ballot[] ballots = new Ballot [Ballot.

    /**
     * Get the number of ballots in this box.
     *
     * @return the number of ballots in this ballot box
     */
   /*@ public normal_behavior
     @   ensures 0 <= \result;
     @   ensures \result == numberOfBallots;
     @   ensures (ballots == null) ==> \result == 0;
     @*/
    public /*@ pure @*/ int size(){
        return numberOfBallots;
    }

  /**
   * The total number of ballots in this ballot box.
   */
    /*@ public invariant 0 <= numberOfBallots;
      @ public invariant numberOfBallots <= Ballot.MAX_BALLOTS;
      @ public constraint \old (numberOfBallots) <= numberOfBallots;
      @*/
    protected /*@ spec_public @*/ int numberOfBallots;

    /**
     * Number of ballots copied from box
     */
    //@ public initially index == 0;
    //@ public invariant index <= size();
    //@ public constraint \old(index) <= index;
    protected /*@ spec_public @*/ int index;

    /**
     * Create an empty ballot box.
     */
    //@ assignable ballots, index, numberOfBallots;
    public /*@ pure @*/ BallotBox(){
```

Refinement Relation

If the input is as we characterized, then we guarantee a correct tally as output.

Overall Correctness Argument

Proof is aggregate modular verification of system's components.

Governments do not trust Verification

Governments think they trust Testing

Automated Testing
that complements
Formal Verification

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```
-- An individual person standing for election
sig Candidate {
 votes:        set Ballot,   -- First preference ballots assigned to this candidate
   transfers:  set Ballot,  -- Second and subsequent preferences received
   surplus:      set Ballot,    -- Ballots tranferred to another candidate election
 wasted:       set Ballot,  -- Ballots non-transferable due to exhaustion of preferences
   outcome:      Event           -- Election result for candidate and associated ballots
} {
   // Non-transferable ballots
   0 < #wasted implies (outcome = WinnerNonTransferable or
                        outcome = QuotaWinnerNonTransferable or
                        outcome = EarlyLoserNonTransferable or
                          outcome = SoreLoserNonTransferable)
   (outcome = WinnerNonTransferable or outcome = QuotaWinnerNonTransferable)
     implies wasted in surplus
   (outcome = EarlyLoserNonTransferable or outcome = SoreLoserNonTransferable)
     implies wasted in votes + transfers
   // Division of ballots into first preferences and transfers
     no b: Ballot | b in votes & transfers
   // Division of ballots into piles for each candidate
     all b: Ballot | b in votes + transfers implies this in b.assignees
   // Selection of surplus ballots for re-distribution
     surplus in votes + transfers
     Election.method = Plurality implies #surplus = 0 and #transfers = 0
     0 < #transfers implies Election.method = STV
   // Calculation of surplus for PR-STV election
   ((outcome = Winner and Election.method = STV) or (
     outcome = SurplusWinner or outcome = WinnerNonTransferable)) implies
     Scenario.quota + #surplus = #votes
      (outcome = Winner or outcome = SurplusWinner or
    outcome = WinnerNonTransferable) implies #transfers = 0
     (outcome = QuotaWinner or outcome = AboveQuotaWinner or
    outcome = QuotaWinnerNonTransferable) implies surplus in transfers
     (outcome = QuotaWinner or outcome = AboveQuotaWinner or
    outcome = QuotaWinnerNonTransferable) implies
        Scenario.quota + #surplus = #votes + #transfers
     0 < #surplus implies (outcome = SurplusWinner or outcome = AboveQuotaWinner or
                 outcome = WinnerNonTransferable or outcome = QuotaWinnerNonTransferable)
     (outcome = EarlyLoser or outcome = TiedEarlyLoser or
    outcome = EarlyLoserNonTransferable) iff
        (this in Scenario.eliminated and
     not (#votes + #transfers < Scenario.threshold))
      // All non-sore losers are at or above the threshold
   outcome = TiedLoser implies Scenario.threshold <= #votes + #transfers
```

e-Voting Test Harness

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   // Selection of surplus ballots for re-distribution
     surplus in votes + transfers
     Election.method = Plurality implies #surplus = 0 and #transfers = 0
     0 < #transfers implies Election.method = STV
   // Calculation of surplus for PR-STV election
   ((outcome = Winner and Election.method = STV) or (
     outcome = SurplusWinner or outcome = WinnerNonTransferable)) implies
     Scenario.quota + #surplus = #votes
      (outcome = Winner or outcome = SurplusWinner or
    outcome = WinnerNonTransferable) implies #transfers = 0
      (outcome = QuotaWinner or outcome = AboveQuotaWinner or
    outcome = QuotaWinnerNonTransferable) implies surplus in transfers
      (outcome = QuotaWinner or outcome = AboveQuotaWinner or
    outcome = QuotaWinnerNonTransferable) implies
        Scenario.quota + #surplus = #votes + #transfers
     0 < #surplus implies (outcome = SurplusWinner or outcome = AboveQuotaWinner or
                 outcome = WinnerNonTransferable or outcome = QuotaWinnerNonTransferable)
     (outcome = EarlyLoser or outcome = TiedEarlyLoser or
    outcome = EarlyLoserNonTransferable) iff
        (this in Scenario.eliminated and
     not (#votes + #transfers < Scenario.threshold))
      // All non-sore losers are at or above the threshold
   outcome = TiedLoser implies Scenario.threshold <= #votes + #transfers
```

e-Voting Test Harness

Table 1 gives – as an example – the numbers from the multi-member constituency of Østjylland (Eastern Jutland).

1.2.2. Step Two: Determining of Passing the Threshold

This step determines which parties are eligible for compensatory seats. This is done by checking if participating parties meet any of three requirements. Thus, the Danish electoral system has not one, but three different electoral thresholds, and parties qualify for participation in the allocation of compensatory seats by any one of them. The three thresholds are:

1. winning a seat directly in any of the ten multi-member constituencies;
2. obtaining in two of the three electoral provinces a number of votes corresponding – at least – to the provincial votes/seat ratio (using in the calculation of these ratios the number of seats in the multi-member constituencies in the electoral provinces in question, excluding the provinces' compensatory seats); or
3. 2 per cent of the valid, national vote.

For parties that do not meet the first requirement (in 2007 it was two of nine participating parties), the relevant numbers are shown in Table 2, which allows a comparison of thresholds (2) and (3), and the votes for the two parties in question in the three electoral provinces as well as nationally.

Experience shows that threshold (3), the 2 per cent rule, is much more important than threshold (2), the vote/seat ratio in two of three electoral provinces. Parties that meet the 2 per cent requirement will often also have met threshold (2) – as was the case in 2007 with the Unity List – while parties below the 2 per cent hurdle almost invariably will not meet any of the other requirements (as shown by the example of the Christian People's Party in 2007, which failed to cross any of the three thresholds). This experience illustrates how Danish political parties are not (any longer) primarily local or provincial in their support patterns.

1.2.3. Step Three: Allocating Compensatory Seats to Parties

This is the decisive step, since it is here that the proportional, overall, national (or upper-tier) allocation of all 175 seats takes place. The calculation (reproduced in Table 3 below) allocates the seats available to parties which have qualified for participation in this allocation in strict proportionality to the number of votes obtained by these parties. The calculation is done on the basis of the so-called pure Hare quota; seats not allocated by the full

Table 2. How the Parties that Failed to Qualify for Seats at Threshold (1) Fared on Threshold (2) and (3). November 13, 2007.

	All of Denmark	Metropolitan Copenhagen	Sealand-Southern Jutland	Northern and Central Jutland
Threshold 2:				
Valid votes per multi-member constituency seat	n.a.	26,906	25,103	25,146
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2 per cent of valid national votes	69,189	-	-	-
The Parties' Votes:				
K. Christian People's Party	30,013	5,513	7,635	16,865
Y. New Alliance	97,295	40,241	30,358	26,696

e-Voting Test Harness

Danish Law

Formally-generated Test Harness

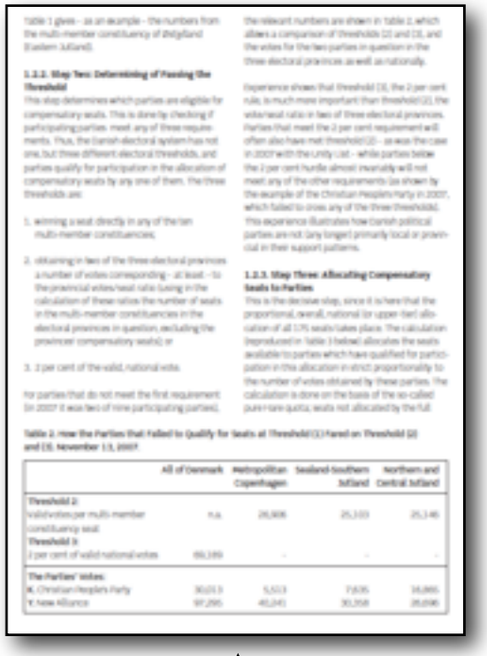

Table 1 gives – as an example – the numbers from the multi-member constituency of Østjylland (Eastern Jutland).

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Experience shows that threshold (3), the 2 per cent rule, is much more important than threshold (2), the votes/seat ratio in two of three electoral provinces. Parties that meet the 2 per cent requirement will often also have met threshold (2) – as was the case in 2007 with the Unity List – while parties below the 2 per cent hurdle almost invariably will not meet any of the other requirements (as shown by the example of the Christian Peoples Party in 2007, which failed to cross any of the three thresholds). This experience illustrates how Danish political parties are not (any longer) primarily local or provincial in their support patterns.

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Table 2. How the Parties that Failed to Qualify for Seats at Threshold (1) Fared on Threshold (2) and (3), November 13, 2007

	All of Denmark	Metropolitan Copenhagen	Sealand-southern Jutland	Northern and Central Jutland
Threshold 2:				
Valid votes per multi-member constituency seat	n.a.	26,808	25,133	25,146
Threshold 3:				
2 per cent of valid national votes	69,199	-	-	-
The Parties' Votes:				
K. Christian Peoples Party	30,013	5,513	7,635	16,865
Y. New Alliance	97,295	45,241	30,358	26,696

Informal EBON

```
class_chart LOGICAL_CLOCK

explanation
  "A logical clock."
query
  "What is the current time for this clock?"
command
  "Advance the clock; update the clock's time."
constraint
  "The time must be non-negative.",
  "Must support concurrent use by multiple clients."
end
```

Formal EBON

```
indexing
  about:        "A logical clock.";
  title:        "TickTockClock";
  author:       "Joe Kiniry";
  copyright:    "Copyright (C) 2007 Joe Kiniry";
  organisation: "School of Computer Science and Informatics, UCD";
  date:         "January 2007";
  version:      "Revision: 11";

static_diagram
component
  deferred class LOGICAL_CLOCK

  feature
    my_time: INTEGER  -- The current time of this clock.

    -- What is the current time of this clock?
    deferred get_logical_time: INTEGER
      -- concurrency: CONCURRENT
      -- modifies: QUERY
      ensure
        Result = my_time;
      end

    deferred advance  -- Advance this clock's time.
      -- concurrency: GUARDED
      -- modifies: my_time
      ensure
        -- This clock's time has monotonically increased.
        old my_time < my_time;
    end

  invariant
    0 <= my_time;

  end -- class LOGICAL_CLOCK

end --component
```

JML

```
/**
 * A logical clock.
 * @title         "TickTockClock"
 * @date          "2007/01/23 18:00:49"
 * @author        "Fintan Fairmichael"
 * @organisation  "CSI School, UCD"
 * @copyright     "Copyright (C) 2007 UCD"
 * @version       "$ Revision: 1.7 $"
 */
public interface LogicalClock {
  // The current time of this clock.
  //@ public model instance \bigint _time;

  //@ public invariant 0 <= _time;

  /**
   * @return What is the current time of this clock?
   * @concurrency CONCURRENT
   */
  //@ ensures \result == _time;
  public /*@ pure @*/ long getLogicalTime();

  /**
   * Advance this clock's time.
   * @concurrency GUARDED
   */
  //@ assignable _time;
  //@ ensures \old(_time) < _time;
  //@ ensures (* _time has been increased. *);
  public void advance();
}
```

```
/** Return a new, freshly allocated indefinite iterator that
 * produces test data of type
 * int[]
 * for testing the method named by the String methodName in
 * a loop that encloses loopsThisSurrounds many other loops.
 * @param methodName name of the method for which this
 *                   test data will be used.
 * @param loopsThisSurrounds number of loops that the test
 *                           contains inside this one.
 */
//@ requires methodName != null && loopsThisSurrounds >= 0;
//@ ensures \fresh(\result);
protected org.jmlspecs.jmlunit.strategies.IndefiniteIterator
    vUnit$_Iter
    (java.lang.String methodName, int loopsThisSurrounds)
{
    return vUnit$_Strategy.iterator();
}

/** The strategy for generating test data of type
 * int[]. */
private org.jmlspecs.jmlunit.strategies.StrategyType
    vUnit$_Strategy
    = new org.jmlspecs.jmlunit.strategies.CloneableObjectAbstractStrategy()
        {
            protected java.lang.Object[] addData() {
                return TestDataGenerator.getIntArrayAsObject();
            }

            //@ also
            //@ requires o$ != null;
            protected Object cloneElement(java.lang.Object o$) {
                int[] down$
                    = (int[]) o$;
                return down$.clone();
            }
        };

/** Return a new, freshly allocated indefinite iterator that
 * produces test data of type
 * election.tally.Ballot
 * for testing the method named by the String methodName in
 * a loop that encloses loopsThisSurrounds many other loops.
 * @param methodName name of the method for which this
 *                   test data will be used.
 * @param loopsThisSurrounds number of loops that the test
 *                           contains inside this one.
 */
//@ requires methodName != null && loopsThisSurrounds >= 0;
//@ ensures \fresh(\result);
```

Java

```
/**
 * A logical clock implementation.
 * @author "Joseph Kiniry"
 */
public class LogicalClockImpl implements LogicalClock {
  /** The current logical time. */
  private long my_time = 0; //@ in _time;
  //@ private represents _time <- my_time;

  public long getLogicalTime() {
    return my_time;
  }

  public void advance() {
    my_time++;
  }
}
```

Refinement Relation

```java
public class Ballot {
    private static final char WHITE_SPACE = ' ';

    /**
     * Maximum possible number of ballots based on maximum population size for a
     * five seat constituency i.e. at most 30,000 people per elected
     * representative.
     *
     * @see "Constitution of Ireland, Article 16, Section 2"
     */
    public static final int MAX_BALLOTS = 150000;

    /**
     * Candidate ID value to use for non-transferable ballot papers
     *
     * @design A special candidate ID value is used to indicate non-transferable
     *         votes i.e., when the list of preferences has been exhausted and
     *         none of the continuing candidates are in the preference list, the
     *         ballot is deemed to be non-transferable.
     * @see <a href="http://www.cev.ie/htm/tenders/pdf/1_2.pdf"> Department Of
     *      Environment and Local Government, Count requirements and Commentary an
     *      Count Rules, section 7, pages 23-27</a>
     */
    public static final int NONTRANSFERABLE = 0;

    /** List of candidates in order of preference */
    // TODO protected invariant preferenceList.owner == this;
    protected/*@ spec_public non_null */int[] preferenceList;

    /** Total number of valid preferences on this ballot paper */
    protected/*@ spec_public @*/int numberOfPreferences;

    /** Position within preference list */
    protected/*@ spec_public @*/int positionInList;

    /**
     * Generate an empty ballot paper for use by a voter.
     */
    /*@ also public normal_behavior
      @   assignable numberOfPreferences, positionInList, preferenceList[*], preferenceList;
      @*/
    public Ballot(final/*@ non_null @*/int[] preferences) {
        numberOfPreferences = preferences.length;
        positionInList = 0;
        int index = 0;
        preferenceList = new int[numberOfPreferences];
        for (int i = 0; i < preferences.length; i++) {
            int preference = preferences[i];
            if (preference != NONTRANSFERABLE
                    && preference != Candidate.NO_CANDIDATE) {
```

```java
/** Return a new, freshly allocated indefinite iterator that
 * produces test data of type
 * int[]
 * for testing the method named by the String methodName in
 * a loop that encloses loopsThisSurrounds many other loops.
 * @param methodName name of the method for which this
 *                   test data will be used.
 * @param loopsThisSurrounds number of loops that the test
 *                           contains inside this one.
 */
//@ requires methodName != null && loopsThisSurrounds >= 0;
//@ ensures \fresh(\result);
protected org.jmlspecs.jmlunit.strategies.IndefiniteIterator
    vint$_Iter
    (java.lang.String methodName, int loopsThisSurrounds)
{
    return vint$_Strategy.iterator();
}

/** The strategy for generating test data of type
 * int[]. */
private org.jmlspecs.jmlunit.strategies.StrategyType
    vint$_Strategy
    = new org.jmlspecs.jmlunit.strategies.CloneableObjectAbstractStrategy()
        {
            protected java.lang.Object[] addData() {
                return TestDataGenerator.getIntArrayAsObject();
            }

            //@ also
            //@ requires o$ != null;
            protected Object cloneElement(java.lang.Object o$) {
                int[] down$
                    = (int[]) o$;
                return down$.clone();
            }
        };

/** Return a new, freshly allocated indefinite iterator that
 * produces test data of type
 * election.tally.Ballot
 * for testing the method named by the String methodName in
 * a loop that encloses loopsThisSurrounds many other loops.
 * @param methodName name of the method for which this
 *                   test data will be used.
 * @param loopsThisSurrounds number of loops that the test
 *                           contains inside this one.
 */
//@ requires methodName != null && loopsThisSurrounds >= 0;
//@ ensures \fresh(\result);
```

90% coverage

Unit Testing from Specs

```
/** Return a new, freshly allocated indefinite iterator that
 * produces test data of type
 * int[]
 * for testing the method named by the String methodName in
 * a loop that encloses loopsThisSurrounds many other loops.
 * @param methodName name of the method for which this
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 * @param loopsThisSurrounds number of loops that the test
 *                           contains inside this one.
 */
//@ requires methodName != null && loopsThisSurrounds >= 0;
//@ ensures \fresh(\result);
protected org.jmlspecs.jmlunit.strategies.IndefiniteIterator
    vint$_Iter
    (java.lang.String methodName, int loopsThisSurrounds)
{
    return vint$_Strategy.iterator();
}

/** The strategy for generating test data of type
 * int[]. */
private org.jmlspecs.jmlunit.strategies.StrategyType
    vint$_Strategy
    = new org.jmlspecs.jmlunit.strategies.CloneableObjectAbstractStrategy()
        {
            protected java.lang.Object[] addData() {
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            }

            //@ also
            //@ requires o$ != null;
            protected Object cloneElement(java.lang.Object o$) {
                int[] down$
                    = (int[]) o$;
                return down$.clone();
            }
        };

/** Return a new, freshly allocated indefinite iterator that
 * produces test data of type
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 * for testing the method named by the String methodName in
 * a loop that encloses loopsThisSurrounds many other loops.
 * @param methodName name of the method for which this
 *                   test data will be used.
 * @param loopsThisSurrounds number of loops that the test
 *                           contains inside this one.
 */
//@ requires methodName != null && loopsThisSurrounds >= 0;
//@ ensures \fresh(\result);
```

90% coverage with only a dozen system tests

Manual System Testing from Law

```
/** Return a new, freshly allocated indefinite iterator that
 * produces test data of type
 * int[]
 * for testing the method named by the String methodName in
 * a loop that encloses loopsThisSurrounds many other loops.
 * @param methodName name of the method for which this
 *                   test data will be used.
 * @param loopsThisSurrounds number of loops that the test
 *                           contains inside this one.
 */
//@ requires methodName != null && loopsThisSurrounds >= 0;
//@ ensures \fresh(\result);
protected org.jmlspecs.jmlunit.strategies.IndefiniteIterator
    vint$_Iter
    (java.lang.String methodName, int loopsThisSurrounds)
{
    return vint$_Strategy.iterator();
}

/** The strategy for generating test data of type
 * int[]. */
private org.jmlspecs.jmlunit.strategies.StrategyType
    vint$_Strategy
    = new org.jmlspecs.jmlunit.strategies.CloneableObjectAbstractStrategy()
        {
            protected java.lang.Object[] addData() {
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            }

            //@ also
            //@ requires o$ != null;
            protected Object cloneElement(java.lang.Object o$) {
                int[] down$
                    = (int[]) o$;
                return down$.clone();
            }
        };

/** Return a new, freshly allocated indefinite iterator that
 * produces test data of type
 * election.tally.Ballot
 * for testing the method named by the String methodName in
 * a loop that encloses loopsThisSurrounds many other loops.
 * @param methodName name of the method for which this
 *                   test data will be used.
 * @param loopsThisSurrounds number of loops that the test
 *                           contains inside this one.
 */
//@ requires methodName != null && loopsThisSurrounds >= 0;
//@ ensures \fresh(\result);
```

for every unique election outcome

System Testing from Law

A Formal Model of Voting

A Parameterized Formal Model of Several Voting Schemes

```
-- An individual person standing for election
sig Candidate {
 votes:        set Ballot,   -- First preference ballots assigned to this candidate
   transfers: set Ballot,  -- Second and subsequent preferences received
   surplus:      set Ballot,    -- Ballots tranferred to another candidate election
 wasted:       set Ballot,  -- Ballots non-transferable due to exhaustion of preferences
   outcome:      Event          -- Election result for candidate and associated ballots
} {
   // Non-transferable ballots
   0 < #wasted implies (outcome = WinnerNonTransferable or
                    outcome = QuotaWinnerNonTransferable or
                    outcome = EarlyLoserNonTransferable or
                      outcome = SoreLoserNonTransferable)
   (outcome = WinnerNonTransferable or outcome = QuotaWinnerNonTransferable)
     implies wasted in surplus
   (outcome = EarlyLoserNonTransferable or outcome = SoreLoserNonTransferable)
     implies wasted in votes + transfers
   // Division of ballots into first preferences and transfers
     no b: Ballot | b in votes & transfers
   // Division of ballots into piles for each candidate
     all b: Ballot | b in votes + transfers implies this in b.assignees
   // Selection of surplus ballots for re-distribution
     surplus in votes + transfers
     Election.method = Plurality implies #surplus = 0 and #transfers = 0
     0 < #transfers implies Election.method = STV
   // Calculation of surplus for PR-STV election
   ((outcome = Winner and Election.method = STV) or (
     outcome = SurplusWinner or outcome = WinnerNonTransferable)) implies
     Scenario.quota + #surplus = #votes
     (outcome = Winner or outcome = SurplusWinner or
    outcome = WinnerNonTransferable) implies #transfers = 0
     (outcome = QuotaWinner or outcome = AboveQuotaWinner or
    outcome = QuotaWinnerNonTransferable) implies surplus in transfers
     (outcome = QuotaWinner or outcome = AboveQuotaWinner or
    outcome = QuotaWinnerNonTransferable) implies
        Scenario.quota + #surplus = #votes + #transfers
     0 < #surplus implies (outcome = SurplusWinner or outcome = AboveQuotaWinner or
                outcome = WinnerNonTransferable or outcome = QuotaWinnerNonTransferable)
     (outcome = EarlyLoser or outcome = TiedEarlyLoser or
    outcome = EarlyLoserNonTransferable) iff
        (this in Scenario.eliminated and
    not (#votes + #transfers < Scenario.threshold))
     // All non-sore losers are at or above the threshold
   outcome = TiedLoser implies Scenario.threshold <= #votes + #transfers
```

Alloy Model

Table 1 gives – as an example – the numbers from the multi-member constituency of Østjylland (Eastern Jutland).

1.2.2. Step Two: Determining of Passing the Threshold

This step determines which parties are eligible for compensatory seats. This is done by checking if participating parties meet any of three requirements. Thus, the Danish electoral system has not one, but three different electoral thresholds, and parties qualify for participation in the allocation of compensatory seats by any one of them. The three thresholds are:

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3. 2 per cent of the valid, national vote.

For parties that do not meet the first requirement (in 2007 it was two of nine participating parties), the relevant numbers are shown in Table 2, which allows a comparison of thresholds (2) and (3), and the votes for the two parties in question in the three electoral provinces as well as nationally.

Experience shows that threshold (3), the 2 per cent rule, is much more important than threshold (2), the vote/seat ratio in two of three electoral provinces. Parties that meet the 2 per cent requirement will often also have met threshold (2) – as was the case in 2007 with the Unity List – while parties below the 2 per cent hurdle almost invariably will not meet any of the other requirements (as shown by the example of the Christian People's Party in 2007, which failed to cross any of the three thresholds). This experience illustrates how Danish political parties are not (any longer) primarily local or provincial in their support patterns.

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K. Christian People's Party	30,013	5,513	7,635	16,865
Y. New Alliance	97,295	40,241	30,358	26,696

```
-- An individual person standing for election
sig Candidate {
 votes:        set Ballot,   -- First preference ballots assigned to this candidate
   transfers:  set Ballot,   -- Second and subsequent preferences received
   surplus:    set Ballot,   -- Ballots tranferred to another candidate election
 wasted:       set Ballot,   -- Ballots non-transferable due to exhaustion of preferences
   outcome:    Event         -- Election result for candidate and associated ballots
} {
   // Non-transferable ballots
   0 < #wasted implies (outcome = WinnerNonTransferable or
                        outcome = QuotaWinnerNonTransferable or
                        outcome = EarlyLoserNonTransferable or
                          outcome = SoreLoserNonTransferable)
   (outcome = WinnerNonTransferable or outcome = QuotaWinnerNonTransferable)
     implies wasted in surplus
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     implies wasted in votes + transfers
   // Division of ballots into first preferences and transfers
     no b: Ballot | b in votes & transfers
   // Division of ballots into piles for each candidate
     all b: Ballot | b in votes + transfers implies this in b.assignees
   // Selection of surplus ballots for re-distribution
     surplus in votes + transfers
     Election.method = Plurality implies #surplus = 0 and #transfers = 0
     0 < #transfers implies Election.method = STV
   // Calculation of surplus for PR-STV election
   ((outcome = Winner and Election.method = STV) or (
     outcome = SurplusWinner or outcome = WinnerNonTransferable)) implies
     Scenario.quota + #surplus = #votes
      (outcome = Winner or outcome = SurplusWinner or
    outcome = WinnerNonTransferable) implies #transfers = 0
      (outcome = QuotaWinner or outcome = AboveQuotaWinner or
    outcome = QuotaWinnerNonTransferable) implies surplus in transfers
      (outcome = QuotaWinner or outcome = AboveQuotaWinner or
    outcome = QuotaWinnerNonTransferable) implies
        Scenario.quota + #surplus = #votes + #transfers
     0 < #surplus implies (outcome = SurplusWinner or outcome = AboveQuotaWinner or
                 outcome = WinnerNonTransferable or outcome = QuotaWinnerNonTransferable)
     (outcome = EarlyLoser or outcome = TiedEarlyLoser or
    outcome = EarlyLoserNonTransferable) iff
        (this in Scenario.eliminated and
     not (#votes + #transfers < Scenario.threshold))
      // All non-sore losers are at or above the threshold
   outcome = TiedLoser implies Scenario.threshold <= #votes + #transfers
```

Law-Alloy Refinement

Rigorous System Test Generation

scenario

candidate

ballot

Core Concepts of Elections

event

method

election

Core Concepts

- candidate
  - votes (set of ballots)
  - transfers (set of ballots)
  - surplus (set of ballots)
  - outcome (event)
- ballot
  - assignees (set of candidates)
  - preferences (sequence of candidates)

Core Concepts

- scenario
  - losers (set of candidates)
  - winners (set of candidates)
  - eliminated (set of candidates)
  - threshold (integer minimum # of votes to not be a sore loser)
  - quota (integer minimum # of votes for an STV or quota winner)

Core Concepts

- event, exactly one of...
  - Winner, QuotaWinner, CompromiseWinner, TiedWinner, TiedLoser, Loser, TiedEarlyLoser, EarlyLoser, TiedSoreLoser, SoreLoser
- election
  - candidates (set of candidates)
  - seats (integer)
  - method (plurality or STV)
  - ballots (integer # of unspoiled ballots)

Generating Scenarios

- goal: generate and characterize every possible non-isomorphism scenario
  - election method, # candidates, # seats
- example outcomes
  - WL or <u>WL</u> in two candidate plurality
  - SSSLLLL<u>LLW</u> with 10 candidates and 1 seat in STV
- scenarios as lemmas
  - “I bet there can’t be an election outcome like this!”

Coupling Systems

- couple Alloy to jUnit
- generate and save system tests in generic format for reuse across implementations
- perform code coverage analysis
- characterize system correctness
- identify suspicious parts of an implementation

Early Results

- first working run two weeks ago
- stopped after several hours to characterize results
- 91% code coverage
- two cases missed in scenario analysis
- zero bugs detected

- expected 2nd run will achieve >99% coverage

Summary of Current Affairs

- formally specified, validated, and verified election tally software systems for US, NL, IE, and DK
- traceable refinement from law—interpreted as concepts, features, and requirements—to specifications, software, and proofs
- automatic verification using ESC/Java2
- automated unit tests with >90% coverage
- manual system tests with >90% coverage
- automated system tests with >99% coverage
- all research and development done in “spare time”

Next Steps

- formal model of elections
  - system model that includes people, parties, bureaucrats, government
- trust-by-design
  - software engineering in the face of an adversarial customer (gov. and citizens)
- logic-based voting scheme
  - couple LFs to implementation

Danish Council for Strategic Research
Programme Commission on
Strategic Growth Technologies

5 years
17M direct
32M total

DemTech

Basin (ETHZ)
Ryan (Lux)

Fredericksberg
Aarhus

Siemens
Aion Assembly

Schürmann
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Markussen

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